Long-range Spin Chains Hamiltonians

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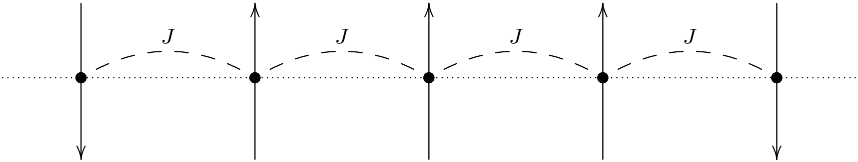
Outline

Quantum spin chains

Nearest neighbor exchange: Heisenberg Hamiltonian

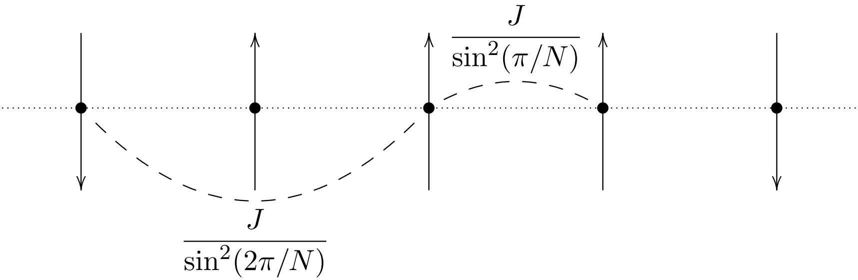
$$\hat{H} = J \sum_j \hat{\vec{S}}_j \cdot \hat{\vec{S}}_{j+1}$$



Quantum spin chains

Long-range exchange: Haldane-Shastry Hamiltonian

$$\hat{H} = J \sum_{\substack{j,k \\ j \neq k}} \frac{\hat{\vec{S}}_j \cdot \hat{\vec{S}}_k}{\sin^2\left(\frac{\pi(j-k)}{N}\right)}$$

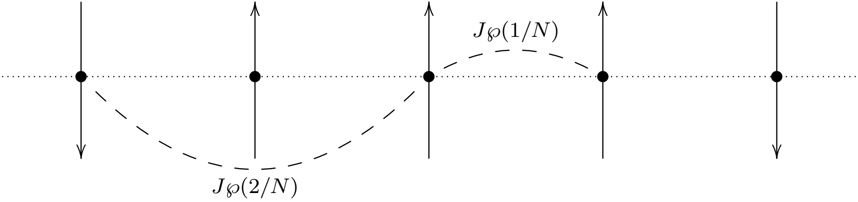


Quantum spin chains

Long-range exchange: Inozemtsev Hamiltonian

$$\hat{H} = J \sum_{\substack{j,k \\ j \neq k}} \wp\left(\frac{j-k}{N}\right) \hat{\vec{S}}_j \cdot \hat{\vec{S}}_k$$

$\wp(z)$ — Weierstrass elliptic function



Quantum spin chains

Idea of integrability

- Big number of independent Hamiltonians $\hat{H}_n$
- $[\hat{H}_n, \hat{H}_m] = 0$
- The Hamiltonians can distinguish every quantum state

The problem to solve

Constructing commuting Hamiltonians for different spin chains with long-range exchange

Integrable systems of particles

Calogero-Moser system of particles

$$H = \sum_i \frac{p_i^2}{2} + \sum_{\substack{i,j \\ i \neq j}} V(q_i - q_j)$$

Interaction potential:

- $V(q) = 1/q^2$ — rational case
- $V(q) = 1/\sin^2(q)$ — trigonometric case
- $V(q) = \wp(q)$ — elliptic case

Integrable systems of particles

Lax representation

$$\dot{p}_j = -\frac{\partial H}{\partial q_j},\ \dot{q}_j = \frac{\partial H}{\partial p_i} \quad \Longleftrightarrow \quad \dot{L}(z) = [L(z), M(z)]$$

$L(z)$ — matrix depends on spectral parameter z and dynamical variables q, p

Integrals of motion

$$\frac{\partial}{\partial t} \operatorname{Tr} L^k(z) = 0$$

R-matrix-valued Lax pair

Modification of Lax pair

Idea: to add quantum degrees of freedom to classical system

- Standard Lax pair in trigonometrical case:

$$L(z) = \sum_{i} p_i E_{ii} + \sum_{\substack{i,j \\ i \neq j}} \big(\cot z + \cot(q_i - q_j) \big) E_{ij}$$

- Lax pair with additional quantum R-matrix:

$$L(z) = \sum_{i} p_i E_{ii} \otimes 1 + \sum_{\substack{i,j \\ i \neq j}} E_{ij} \otimes R^{z}_{ij}(q_i - q_j)$$

R-matrix-valued Lax pair

Spin chain Hamiltonians

Algorithm: (Hypothesis)

- Computing n-th Hamiltonian of Calogero system
- Constructing n-th R-matrix-valued Lax pair

$$\frac{\partial}{\partial t_n} L(z) = \{H_n, L(z)\} = [L(z), M_n(z)]$$

- Choosing the equidistant points: $p_j = 0$, $q_j = j/N$
- Calculating trace over the auxiliary (first) space
- ???
- PROFIT

First results

- Third Hamiltonian for long-range spin chains

$$H_3 = \sum_{\substack{i,j,k \\ i\neq j\neq k\neq i}} [F^0_{ij}(\frac{i-j}{N}), r_{jk}(\frac{j-k}{N})]$$

$$F^0_{ij}(q) = \frac{\partial}{\partial z} R^z_{ij}(q)|_{z=0}, \quad r_{ij}(q) = \lim_{z\to 0}(R^z_{ij}(q) - \frac{1}{z})$$

First results

- Different R-matrices represents different spin models
  - $R^z(q) = P/\sin(z) + P/\sin(q)$ — Haldane-Shastry
  - $R^z(q) = P \cdot \phi(z, q)$ — Inozemtsev
    (ϕ — one of the elliptic functions, P — permutation operator)
  - $R^z(q)$ — XXZ R-matrix — new **anisotropic** long-range model
- For both isotropic cases the result is proven $\forall N$
- Anisotropic case — only if $N \leq 5$