Hawking Radiation & Secularly Growing Loop Corrections

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Outline

Setup of the problem

We consider a massive scalar field on a thin-shell collapse background in 4D.

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{m^2}{2} \phi^2 - \lambda \phi^4 - \dots$$

$$ds^2 = \begin{cases} ds_+^2 = \left(1 - \frac{r_g}{r}\right) dt_+^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 d\Omega_2^2, r > R(t) \\ ds_-^2 = dt_-^2 - dr^2 - r^2 d\Omega_2^2, r < R(t) \end{cases}$$

Where $R(t)$ is a trajectory of shell. It is assumed, that initially the shell didn't move and was at radius R_0 and after some moment it starts to move.

$$R(t) = \begin{cases} R_0, t \leq 0 \\ R(t), t > 0 \end{cases}$$

We want to understand what is happening with scalar field during collapse

Gravitational preliminary

We have to sew metric inside and outside the shell. Before collapse appeared it leads to the equality

$$ds^2_{sh,-} = ds^2_{sh,+} \Rightarrow t_- = \sqrt{1 - \frac{r_g}{R_0}} t_+$$

The thin-shell collapses according to the following law(that is obtained by using Einstein equation)

$$R(t_+) \approx r_g + (R_0 - r_g) e^{-\frac{t_+}{r_g}}, R(t_-) = R_0 - v t_-$$

And we get how to express t_- through t_+

$$t_- \approx \frac{R_0 - r_g}{v} \left(1 - e^{-\frac{t_+}{r_g}} \right)$$

The trajectory of shell

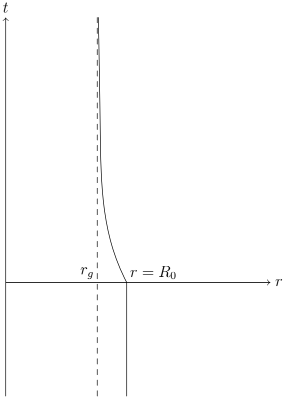


Figure: The trajectory of thin-shell according external observer

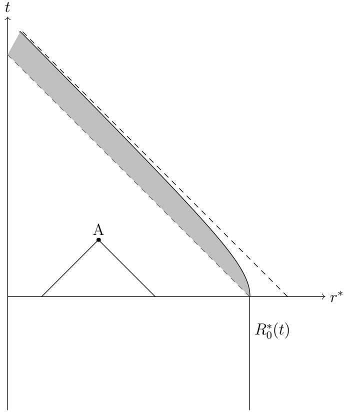


Figure: The trajectory of thin-shell in tortoise coordinates

EOM for free harmonics

The simplest case is a free scalar field. Also we consider only in-harmonics, that diagonalize Hamiltonian, when shell doesn't move.

EOMs for harmonics are

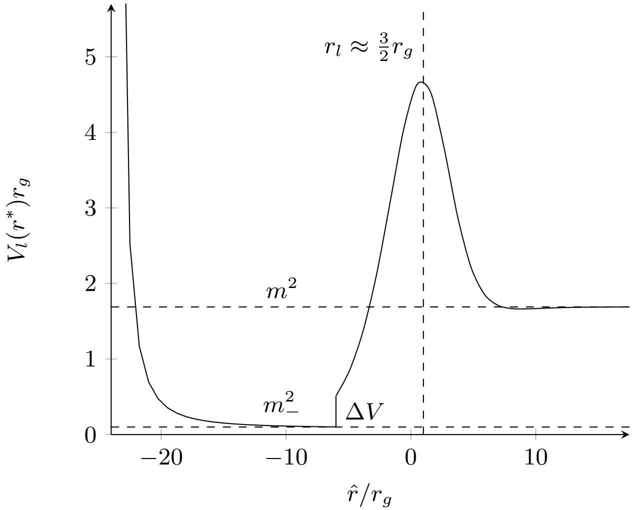
$$\phi(t,r,\theta,\alpha) = \sum_{l,m} Y_{l,m}(\theta,\alpha)\phi_{l,m}(r,t)$$

$$\begin{cases} \left[\partial_{t_-}^2 - \partial_r^2 + m^2 + \frac{l(l+1)}{r^2}\right](r\phi_l) = 0, r < R(t) \\ \left[\partial_{t_+}^2 - \partial_{r^*}^2 + (1 - \frac{r_g}{r})\left(m^2 + \frac{l(l+1)}{r^2} + \frac{r_g}{r^3}\right)\right](r\phi_l) = 0, r > R(t) \end{cases}$$

$$r^* = r + r_g \log\left(\frac{r}{r_g} - 1\right)$$

$$V(r_*) = \begin{cases} \left(1 - \frac{r_g}{r_*}\right)\left(m^2 + \frac{l(l+1)}{r_*^2} + \frac{r_g}{r_*^3}\right), r < R \\ m^2 + \frac{l(l+1)}{r_*^2}, r > R \end{cases}$$

Qualitative plot of potential $V(\hat{r})$



Spectrum of the in-theory

In order to find a spectrum we consider the following dependence on t_+ for harmonics

$$\phi_l \propto e^{-i\omega t} \Rightarrow \left[-\partial^2_{r_*} + V(r^*)\right] \phi_l = \omega^2 \phi_l$$

It reminds Schroedinger equation.
We conclude that spectrum of theory is

1. Discrete in the range $m\left(1 - \frac{r_g}{R_0}\right) < \omega_n < m$

$$\omega_n^2 \approx m_-^2 + \frac{n^2}{L^2}, L \sim r_g$$

2. Continuous for $\omega > m$

The further evolution of harmonics

Under thin-shell, free field doesn't know about collapse and hence it depends on time as follows

$$\phi_l \propto e^{-i\omega t_-} \Rightarrow \phi_l(R, t_+) \propto e^{-i\omega \frac{R_0 - r_g}{v} e^{-\frac{t_+}{r_g}}}$$

We assume, that $v \approx 1$. Near horizon the general solution of free field equation is

$$\phi_l = f(\underbrace{t + r_*}_{u}) + g(\underbrace{t - r_*}_{v})$$

By imposing the condition of continuity we get harmonics outside of shell

$$\phi_l(u, v) = \frac{1}{\sqrt{2\omega} r_g} e^{i\omega r_g e^{-\frac{u}{2r_g}}} + \frac{1}{\sqrt{2\omega} r_g} e^{-i\omega v}$$

Hawking radiation

Let us calculate fluxes of J_u and J_v for stationary observer

$$J_u = \int r^2 d\Omega_2 \left(\partial_u \phi\right)^2 , J_v = \int r^2 d\Omega_2 \left(\partial_v \phi\right)^2$$

Straightforward calculation shows that

$$J_v = \int \frac{d\omega}{2\pi} \frac{\omega}{2}, J_u = \int \frac{d\omega}{2\pi} \frac{\omega}{2} + \int \frac{d\omega}{2\pi} \omega n(\omega), n(\omega) = \frac{1}{e^{4\pi r_g \omega} - 1}$$

And total flux is a "thermal" one.

$$J_{st.} = \int \frac{d\omega}{2\pi} \frac{\omega}{e^{4\pi r_g \omega} - 1}$$

Loop corrections

By using such harmonics we can calculate loop corrections to some quantity. But because situation is non-stationary we have to use Schwinger Keldysh formalism. It contains three propagators $D^{K,R,A}$.

The most interesting one is D^K because it contains information about level population $D^K \propto f_\omega f_{\omega'} n_{\omega,\omega'}$, $n_{\omega,\omega'} = \langle a^\dagger_\omega a_{\omega'} \rangle$. One-loop perturbation level gives

$$n_{\omega,\omega'}(t) = \lambda^2 \int_{t_1 \le t} d^4x_1 \int_{t_2 \le t} d^4x_2 f_\omega(x_1) f^*_{\omega'}(x_2) \prod_{i=1}^{3} \int \frac{d\omega_i}{2\pi} f_{\omega_i}(x_1) f^*_{\omega_i}(x_2)$$

$$n_{\omega,\omega'}(t) \propto \lambda^2 (t - t_0)$$

Where t_0 is a moment when shell starts to move. Such linear growth corresponds to particle creation.

Discussion

To understand the meaning of it we have to sum up the leading contributions from all level of perturbation series.

The summation leads to the constant shift of the number of particles. The corresponding solution is

$$n_\omega(u,\theta,\phi) = \frac{1}{\exp\left(\frac{\omega}{T(u,\theta,\phi)}\right) - 1}$$

It modifies Hawking radiation and can resolve Information Paradox. Indeed, because temperature depends on u, θ, ϕ we can extract a lot of information out of this.

Conclusion

1. The spectrum of scalar field theory was considered in thin-shell metric background.
2. The flux that is seen by stationary observers was obtained.
3. Linearly growing with time loop corrections was obtained for scalar field in gravitational collapse background.

Thank you for your attention!