Chiral Waves in Chiral Kinetic Theory

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Plan

- What do we study: hot system of massless fermions in external magnetic field and under rotation
- How do we study : basic equations
- How do we solve these equations
- Results
- Discussion and conclusion

What do we study

- weakly interacting massless fermions
- left and right do not mix
- high temperature
- particles and antiparticles
- external magnetic field $\mathbf{B}$ and rotation $\mathbf{\Omega}$, weak compared to temperature

Kinetic equation

$$\frac{\partial f}{\partial t} + \dot{\mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{x}} + \dot{\mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{p}} = St[f]$$

$f(t, x, p)$ is distribution function, $St[f]$ is collision integral, $\dot{\mathbf{x}}$ and $\dot{\mathbf{p}}$ are given by the equations of motion

Equations of motion

$$\dot{\mathbf{x}} = \hat{\mathbf{p}} \; + \; \dot{\mathbf{p}} \times \mathbf{b}$$

- Here $\mathbf{b} = \dfrac{\hat{\mathbf{p}}}{2p^2}$ is the curvature of Berry's connection in momentum space - the crucial ingredient
- It allows to reconstruct the expressions for chiral anomaly, Chiral Magnetic Effect, etc. through chiral kinetic theory

$$\dot{\mathbf{p}} = \dot{\mathbf{x}} \times \mathbf{B}'$$

- Here $\mathbf{B}' = \mathbf{B} + 2p\boldsymbol{\Omega}$ plays the role of effective magnetic field
- $\dot{\mathbf{x}}$ and $\dot{\mathbf{p}}$ are expressed via each other

Resolved equations of motion

$$\sqrt{G}\dot{\mathbf{x}} = \hat{\mathbf{p}} + \frac{\mathbf{B}'}{2p^2}$$
$$\sqrt{G}\dot{\mathbf{p}} = \mathbf{p} \times \mathbf{B}'$$

Here $\sqrt{G} = 1 + \frac{\mathbf{B}' \cdot \mathbf{p}}{p^2}$ modifies phase space volume due to the interplay between Berry connection and effective magnetic field.

Equations of motion for al kinds of particles

$$\sqrt{G_{\pm R}}\dot{\mathbf{x}} = \hat{\mathbf{p}} + \tfrac{\mathbf{B}'_{\pm}}{2p^2}$$

$$\sqrt{G_{\pm L}}\dot{\mathbf{x}} = \hat{\mathbf{p}} - \tfrac{\mathbf{B}'_{\pm}}{2p^2}$$

$$\sqrt{G_{\pm R}}\dot{\mathbf{p}} = \sqrt{G_{\pm L}}\dot{\mathbf{p}} = \pm\mathbf{p} \times \mathbf{B}'_{\pm}$$

Here $\sqrt{G_{\pm R}} = 1 + \dfrac{\mathbf{B}'_{\pm} \cdot \mathbf{p}}{p^2}$ and $\sqrt{G_{\pm L}} = 1 - \dfrac{\mathbf{B}'_{\pm} \cdot \mathbf{p}}{p^2}$

$\pm$ - particles and antiparticles, R/L - right and left

How to solve kinetic equation

- linearise : $f = f_0 + \delta f$, where $f_0 = \frac{1}{e^{\beta(p-\mu)}+1}$
- go to Fourier space for δf
- integrate kinetic equation over the momentum space with the right measure, modified by the factors of $\sqrt{G}$
- make use of conservation laws: number of particles and energy to get rid of collision integrals
- consider hydrodynamic modes, corresponding to infinitesimal shifts of chemical potential and temperature

Hydrodynamic modes

$$\delta f_{R\pm} = \frac{\partial f_{0\pm}}{\partial p}(\pm h_1 + \beta p\ h_3)$$

$$\delta f_{L\pm} = \frac{\partial f_{0\pm}}{\partial p}(\pm h_2 + \beta p\ h_3)$$

- Here roughly h_1 corresponds to $\delta\mu_R$, h_2 to $\delta\mu_L$ and h_3 to δT and in the hydrodynamic limit $\mathbf{k} \to 0$ are independent on $\mathbf{p}$
- There finally is a set of linear equations on h_i that determines the dispersion relation

Results: general

- In the general case of nonvanishing $\mathbf{B}$, $\mathbf{\Omega}$ and μ the answer is complicated
- So it might be useful to analyse different special cases separately first

Results : Chiral Magnetic Wave

- $\mathbf{B} \neq 0$, $\mathbf{\Omega} = 0$, $\mu = 0$
- The wave propagates along the magnetic field with the velocity

$$v_{CMW} = \frac{B}{4\pi^2\chi}$$

- Here $\chi = \dfrac{T^2}{6} = \dfrac{\partial n_0}{\partial \mu}|_{\mu=0}$ is charge susceptibility

Results: Chiral Heat Wave

- $\mathbf{B} = 0$, $\mathbf{\Omega} \neq 0$, $\mu = 0$
- The wave propagates along the vorticity with the velocity

$$v_{CHW} = \frac{\Omega}{\pi T}\sqrt{\frac{5}{14}}$$

- Equivalently, using χ and $C_V = \dfrac{14\pi^2 T^3}{15} = \dfrac{\partial \epsilon}{\partial T}$ - heat capacity we may rewrite as

$$v_{CHW} = \frac{\Omega}{3}\sqrt{\frac{T^3}{2C_V\chi}}$$

Results: Mixed Chiral Heat and Vortical Wave

- $\mathbf{B} = 0$, $\mathbf{\Omega} \neq 0$, $\mu \neq 0$, $\mu \ll T$
- The wave propagates along the vorticity with the velocity

$$v_{HV} = \Omega \sqrt{\frac{T^3}{18 C_V \chi} + \mu^2 \left(\frac{1}{4\pi^4 \chi^2} + \frac{T}{6\pi^2 C_V \chi} + \frac{T^2}{C_V^2} - \frac{5T}{4\pi^2 \chi C_V} \right)}$$

- It is valid up to the order quadratic in μ/T

Results: Mixed Chiral Heat and Magnetic Wave

- $\mathbf{B} \neq 0$, $\mathbf{\Omega} \neq 0$, $\mu = 0$
- The wave propagates in the plane where $\mathbf{B}$ and $\mathbf{\Omega}$ are lying
- There are two mutually orthogonal directions for which the velocity is maximal and minimal and is equal

$$v^2_{MH\pm} =$$

$$\frac{v^2_{CHW} + v^2_{CMW}}{2} \pm \frac{1}{2}\sqrt{v^4_{CHW} + v^4_{CMW} + 2v^2_{CHW}v^2_{CMW}\cos 2\phi}$$

- Here ϕ is the angle between $\mathbf{B}$ and $\mathbf{\Omega}$
- For arbitrary direction velocity of the wave is expressed through $v_{MH\pm}$ but the expression is not quite attractive

Results : Mixed Chiral Heat,Magnetic and Vortical Wave

- $\mathbf{B} \neq 0$, $\mathbf{\Omega} \neq 0$, $\mu \neq 0$, $\mu \ll T$
- The wave propagates in the plane where $\mathbf{B}$ and $\mathbf{\Omega}$ are lying
- There are two mutually orthogonal directions for which the velocity is maximal and minimal and the expression is not quite attractive, so we don't give it here

Conclusion

- Chiral kinetic theory may be used to obtain dispersion relations of different chiral waves
- The results obtained coincide with the ones obtained in hydrodynamic approach
- In general, there also exists Chiral Vortical Wave and its' mixing with Chiral Magnetic Wave, but they can't be investigated in kinetic theory for hot plasma because at high temperatures Chiral Heat Wave dominates in presence of vorticity

THANK YOU FOR YOUR ATTENTION